[13.0017]

1) $P^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

2) $P^{-1}AP = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 4 \end{pmatrix} = D$

donc $A = PDP^{-1}$

$A^{27} = PDP^{-1}\underbrace{.PDP^{-1}}_{I}\underbrace{.PDP^{-1}}_{I}.PDP^{-1} \dots PDP^{-1}$

$= PD^{27}P^{-1}$

$$= \begin{pmatrix} 2^{26}(2^{27}+3^{27}) & 2^{26}(4^{27}-3^{27}) & 2^{53}(2^{27}-1) \\ 2^{26}(2^{27}-3^{27}) & 2^{26}(3^{27}+4^{27}) & 2^{53}(2^{27}-1) \\ 2^{26}(3^{27}-2^{27}) & 2^{26}(4^{26}-3^{26}) & 2^{53}(2^{27}+1) \end{pmatrix}$$