[11.0102]

$f(x) = 3x - 4x^3$

f est définie et continue sur $\mathbb{R}$.

$f'(x) = 3 - 12x^2$

$f'(x) = 0 \iff 3 - 12x^2 = 0$

$\iff 3 = 12x^2$

$\iff x^2 = \frac{3}{12}$

$\iff \begin{cases} x = +\sqrt{\frac{3}{12}} = +\sqrt{\frac{1}{4}} = \frac{1}{2} \\ x = -\sqrt{\frac{3}{12}} = -\sqrt{\frac{1}{4}} = -\frac{1}{2} \end{cases}$

x	$-\infty$		$-\frac{1}{2}$		$\frac{1}{2}$		$+\infty$
f'		$-$	0	$+$	0	$-$	
f	$+\infty$	↘	-1	↗	1	↘	$-\infty$

$f'(0) = 3 > 0$

$f'(1) = 3 - 12 = -9 < 0$

$f'(-1) = 3 - 12 = -9 < 0$

$f\left(-\frac{1}{2}\right) = -\frac{3}{2} + \frac{4}{8}$

$= -\frac{3}{2} + \frac{1}{2}$

$= -1$

$f\left(\frac{1}{2}\right) = \frac{3}{2} - \frac{4}{8}$

$= \frac{3}{2} - \frac{1}{2}$

$= 1$

$f(x) = 3x - 4x^3 = x^3\left(\frac{3}{x^2} - 4\right)$

$\lim\limits_{x \to +\infty} \frac{3}{x^2} = \lim\limits_{x \to -\infty} \frac{3}{x^2} = 0$

$\lim\limits_{x \to +\infty} \left(\frac{3}{x^2} - 4\right) = \lim\limits_{x \to -\infty} \left(\frac{3}{x^2} - 4\right) = -4$

donc $\lim\limits_{x \to +\infty} f(x) = -\infty$ et $\lim\limits_{x \to -\infty} f(x) = +\infty$