04.0016.C

[redacted]

$$A = \begin{pmatrix} 0 & 2 & -1 \\ 3 & -2 & 0 \\ -2 & 2 & 1 \end{pmatrix}$$

$$P_A(\lambda) = \begin{vmatrix} -\lambda & 2 & -1 \\ 3 & -2-\lambda & 0 \\ -2 & 2 & 1-\lambda \end{vmatrix}$$

$$= (-1)^{1+3} \times (-1) \times \left(3 \times 2 - 2(2+\lambda)\right)$$

$$+ (-1)^{3+3} \times (1-\lambda) \times \left(\lambda(2+\lambda) - 2 \times 3\right)$$

$$= \left(2(2+\lambda) - 3 \times 2\right) + (1-\lambda)\left(\lambda(2+\lambda) - 2 \times 3\right)$$

$$= (4+2\lambda-6) + (1-\lambda)(\lambda^2+2\lambda-6)$$

$$= (2\lambda-2) + (1-\lambda)(\lambda^2+2\lambda-6)$$

$$= (\lambda-1)\left(2 - (\lambda^2+2\lambda-6)\right)$$

$$= (\lambda-1)(-\lambda^2-2\lambda+8)$$

$$= (\lambda-1)(\lambda+4)(2-\lambda)$$

Donc $\mathrm{Spec}(A) = \{1, 2, -4\}$

Donc A est diagonalisable

$$AX = 2X \iff \begin{cases} 2y - z = 2x \\ 3x - 2y = 2y \\ -2x + 2y + z = 2z \end{cases}$$

$$\iff \begin{cases} z = 2y - 2x \\ 3x = 4y \end{cases}$$

$$\iff \begin{cases} 2z = 4y - 4x \\ 3x = 4y \end{cases}$$

$$\iff \begin{cases} 2z = 3x - 4x \\ 3x = 4y \end{cases}$$

$$\iff \begin{cases} 2z = -x \\ 3x = 4y \end{cases}$$

$$\iff 3x = 4y = -6z$$

$$\iff X = \begin{pmatrix} -2z \\ -\frac{3}{2}z \\ z \end{pmatrix}$$

$$\iff X = \frac{z}{2} \begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix}$$

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$$AX = X \iff \begin{cases} 2y - z = x \\ 3x - 2y = y \\ -2x + 2y + z = z \end{cases}$$

$$\iff \begin{cases} x + z = 2y \\ x = y \end{cases}$$

$$\iff x = y = z$$

$$\iff X = \begin{pmatrix} x \\ x \\ x \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$AX = -4X \iff \begin{cases} 2y - z = -4x \\ 3x - 2y = -4y \\ -2x + 2y + z = -4z \end{cases}$$

$$\iff \begin{cases} z = 2y + 4x \\ 3x = -2y \\ 2y = -5z + 2x \end{cases}$$

$$\iff \begin{cases} 3x = -2y \\ z = x \end{cases}$$

$$\iff X = \begin{pmatrix} x \\ -\frac{3}{2}x \\ x \end{pmatrix} = -\frac{1}{2}x \cdot \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}$$

d'où $P = \begin{pmatrix} -4 & 1 & -2 \\ -3 & 1 & 3 \\ 2 & 1 & -2 \end{pmatrix}$ et $D = \begin{pmatrix} 2 & & 0 \\ & 1 & \\ 0 & & 4 \end{pmatrix}$

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$$B = \begin{pmatrix} 0 & 3 & 2 \\ -2 & 5 & 2 \\ 2 & -3 & 0 \end{pmatrix}$$

$$P_B(\lambda) = \begin{vmatrix} -\lambda & 3 & 2 \\ -2 & 5-\lambda & 2 \\ 2 & -3 & -\lambda \end{vmatrix} \quad L_1 \leftarrow L_1 + L_3$$

$$= \begin{vmatrix} 2-\lambda & 0 & 2-\lambda \\ -2 & 5-\lambda & 2 \\ 2 & -3 & -\lambda \end{vmatrix}$$

$$= (-1)^{1+1} \times (2-\lambda) \times \left((5-\lambda)(-\lambda) + 2 \times 3\right)$$
$$+ (-1)^{1+3} \times (2-\lambda) \times \left((-2)\times(-3) - 2 \times (5-\lambda)\right)$$
$$= (2-\lambda) \times \left(-\lambda(5-\lambda) + 6 + 6 - 2 \times (5-\lambda)\right)$$
$$= (2-\lambda) \times \left((5-\lambda)(-\lambda-2) + 12\right)$$
$$= (2-\lambda) \times (2-\lambda)(1-\lambda)$$

Donc $Spec(B) = \{2, 1\}$

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$$BX = 2X \iff \begin{cases} 3y + 2z = 2x \\ -2x + 5y + 2z = 2y \\ 2x - 3y = 2z \end{cases}$$

$$\iff 2x = 3y + 2z$$

$$\iff X = \begin{pmatrix} \frac{3}{2}y + z \\ y \\ z \end{pmatrix} = \frac{1}{2}y \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$BX = X \iff \begin{cases} 3y + 2z = x \\ -2x + 5y + 2z = y \\ 2x - 3y = z \end{cases}$$

$$\iff \begin{cases} x = 3y + 2z \\ 2x = 4y + 2z \\ 2x = 3y + z \end{cases}$$

$$\iff \begin{cases} x = 3y + 2z \\ 0 = y + z \end{cases}$$

$$\iff \begin{cases} y = -z \\ x = y \end{cases}$$

$$\iff X = \begin{pmatrix} x \\ x \\ -x \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

Donc A est diagonalisable et

$$P = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \quad \text{et} \quad D = \begin{pmatrix} 2 & & 0 \\ & 2 & \\ 0 & & 1 \end{pmatrix}$$

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$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix}$$

$$P_c(\lambda) = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 1 & -1 & 2-\lambda \end{vmatrix}$$

$$= (-1)^{1+1} \times (1-\lambda) \times (1-\lambda) \times (2-\lambda)$$

$$= (1-\lambda)^2 \times (2-\lambda)$$

$$\mathrm{Spec}(C) = \{1, 2\}$$

$$CX = X \iff \begin{cases} x = x \\ y = y \\ x - y + 2z = z \end{cases}$$

$$\iff y = x + z$$

$$\iff X = \begin{pmatrix} x \\ x+z \\ z \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$CX = 2X \iff \begin{cases} x = 2x \\ y = 2y \\ x - y + 2z = 2z \end{cases}$$

$$\iff x = y = 0$$

$$\iff X = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} = z \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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Donc C est diagonalisable.

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{et} \quad D = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 2 \end{pmatrix}$$